The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Final Exam

Date: August 15, 2016 Course: EE 313 Evans

Name: ____
Last, First

- The exam is scheduled to last three hours.
- Open books and open notes. You may refer to your homework assignments and homework solution sets.
- **Power off all cell phones**
- You may use any standalone calculator or other computing system, i.e. one that is not connected to a network.
- Please do not wear hats or headphones during the exam.
- All work should be performed on the exam itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise.**

Problem	Point Value	Your score	Topic
1	8		Continuous-Time Fourier Transform
2	12		Continuous-Time Frequency Response
3	8		Discrete-Time Fourier Transform
4	12		Discrete-Time Frequency Response
5	12		Discrete-Time Filter Design
6	12		Circuit Analysis
7	12		Convolution
8	12		Averaging Filters
9	12		Stability
Total	100		

Problem 1. Continuous-Time Fourier Transform. *8 points*.

The continuous-time Fourier transform transforms a continuous-time function $x(t)$ into a function $X(\omega)$ of a real-variable ω as follows:

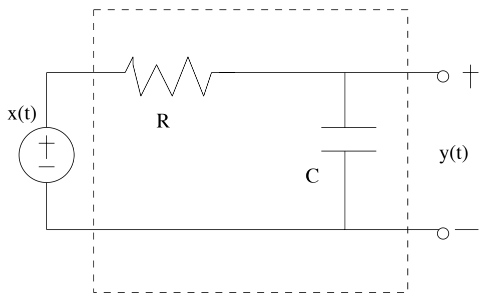
$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$$

(a) Using only the continuous-time Fourier transform definition above, find the Fourier transform of the continuous-time impulse $\delta(t)$. *4 points*.

(b) Using only the continuous-time Fourier transform definition above, find the Fourier transform of a causal exponential signal $\exp(-t / \tau)\, u(t)$ where τ is the time constant in units of seconds where $\tau > 0$ and $u(t)$ is the unit step function. *4 points*.

Problem 2. Continuous-Time Frequency Response. *12 points*.

Consider the following analog continuous-time circuit with input $x(t)$ and output $y(t)$:



Analyze the circuit for $t > 0^-$ given that the initial voltage across the capacitor is 0 V.

(a) Give a formula for the frequency response of the circuit. *3 points*.

(b) Plot the magnitude of the frequency response of the circuit. *3 points.*

(c) What is the frequency selectivity of the circuit? Lowpass, highpass, bandpass, bandstop, allpass or notch. Why? *3 points*.

(d) What is the bandwidth of the circuit? *3 points*.

Problem 3. Discrete-Time Signals. *8 points*.

The unilateral z-transform transforms a discrete-time function $x[n]$ into a function $X[z]$ of a complex-variable z as follows:

$$X[z] = \sum_{n=0}^{\infty} x[n] z^{-n}$$

The discrete-time Fourier transform transforms a discrete-time function $x[n]$ into a function $X_{freq}(\omega)$ of a real-variable ω as follows

$$X_{freq}(\omega) = \sum_{n=0}^{\infty} x[n] e^{-j\omega n}$$

Sometimes, the z-transform can be used to compute the discrete-time Fourier transform.

(a) Use the z-transform (definition, lookup tables, etc.) and appropriate additional work to find the discrete-time Fourier transform of the discrete-time impulse $\delta[n]$. *4 points*.

(b) Use the z-transform (definition, lookup tables, etc.) and appropriate additional work to find the discrete-time Fourier transform of a causal exponential signal $a^n\, u[n]$ where a is a complex-valued constant and $|a| < 1$. *4 points*.

Problem 4. Discrete-Time Systems. *12 points*.

Consider a causal discrete-time linear time-invariant system with input $x[n]$ and output $y[n]$ being governed by the following difference equation

$$y[n] = (2\, r \cos \omega_0)\, y[n-1] - r^2\, y[n-2] + x[n]$$

where r is a real number $0 < r < 1$.

(a) Draw a block diagram for the difference equation. *3 points*.

(b) Please state all initial conditions. Please give values for the initial conditions to satisfy the stated system properties. *3 points*.

(c) Find the equation for the transfer function $H[z]$ in the z-domain, including the region of convergence. *3 points*.

(d) Find a formula for the frequency response for the system. *3 points*.

Problem 5. Discrete-Time Filter Design. *12 points*.

We are going to design parameters for the system in Problem 4.

The system is a causal discrete-time linear time-invariant system with input $x[n]$ and output $y[n]$ being governed by the following difference equation

$$y[n] = (2\, r \cos \omega_0)\, y[n-1] - r^2\, y[n-2] + x[n]$$

where r is a real number $0 < r < 1$. The poles are located at $z = r \exp(j\, \omega_0)$ and $z = r \exp(-j\, \omega_0)$.

The signal $x[n]$ results from sampling a continuous-time signal $x(t)$ at a sampling rate of f_s.

The filter should pass the continuous-time frequency f_0 in Hz and attenuate as many of the other frequencies as possible.

(a) What are all of the posible values of f_0? *3 points*.

(b) Determine the value of ω_0. *3 points*.

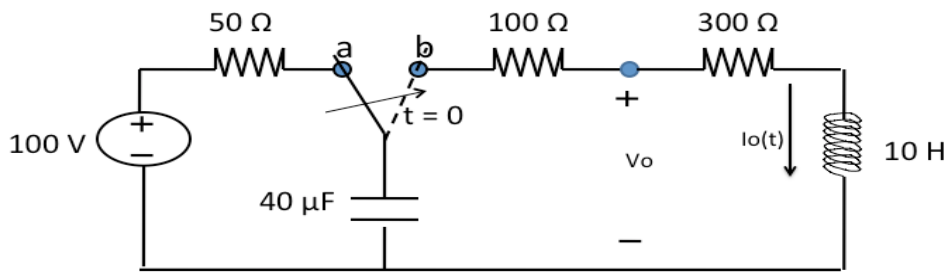
(c) Determine the value of r where $0 < r < 1$. *3 points*.

(d) If $r = 1$, give a formula for the output $y[n]$ if $x[n] = \delta[n]$. *3 points*.

Problem 6. Circuit Analysis. *12 points.*

The switch in the circuit below has been in position 'a' for a long time.

At t=0, the switch is moved to position 'b'.



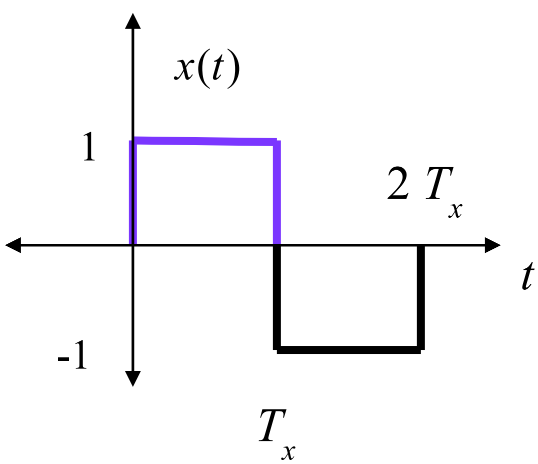
Determine $v_o(t)$ by taking the Laplace transform of the circuit, and then solving for the time-domain expression.

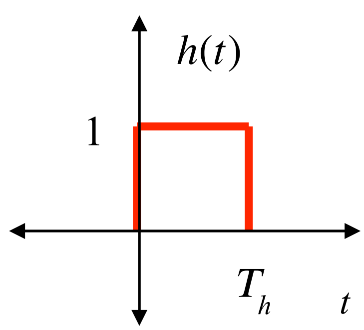
Problem 7. Convolution. *12 points.*

Please solve the following two convolution problems and explain the method you have chosen to use.

(a) Convolve the unit step function $u(t)$ with the signal $\delta(t) - \delta(t - T)$ where $T > 0$. Plot all three signals: $u(t)$ and $\delta(t) - \delta(t - T)$ and their convolution. *6 points.*

(b) Convolve the following continuous-time signals, assuming that $T_h < T_x$. Plot the result. *6 points.*





Problem 8. Averaging Filters. *12 points*.

An averaging filter is used to reduce noise and smooth out data from one value to the next.

A two-coefficient averaging filter with input $x[n]$ and output $y[n]$ is

$$y[n] = \frac{1}{2}x[n] + \frac{1}{2}x[n-1] = \frac{x[n] + x[n-1]}{2}$$

That is, the output is the average value of the current input value and the previous input value.

The three-coefficient averaging filter with input $x[n]$ and output $y[n]$ is

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2] = \frac{x[n] + x[n-1] + x[n-2]}{3}$$

Please answer the following questions about the **three-coefficient averaging filter**.

(a) What are the initial conditions of the averaging filter? What values should they have in order for the averaging filter to have the system properties of linearity and time-invariance? *3 points*.

(b) Give a formula for the impulse response of the averaging filter. Plot the impulse response. Is the impulse response of finite duration or infinite duration? *3 points.*

(c) What is the transfer function in the z-domain? Please include the region of convergence. *3 points.*

(d) Plot the magnitude response. What kind of frequency selectivity does this filter have? Lowpass, highpass, bandpass, bandstop, allpass or notch? *3 points.*

Problem 9. Stability. *12 points.*

Consider a system with input $x(t)$ and output $y(t)$ governed by the differential equation

$$y''(t) + y(t) = x'(t) - x(t)$$

for $t > 0^-$.

(a) What are the characteristic roots of the system? *3 points*.

(b) What are the initial conditions of the system? *3 points*.

(c) Assume that the initial conditions are not zero. Is the zero-input response asymptotically unstable, marginally stable or stable? *3 points.*

(d) Assume that the initial conditions are zero. What is the transfer function in the Laplace domain? Is the system bounded-input bounded-output (BIBO) stable? Why or why not? *3 points*.